5. Coordinate Geometry

1 mark Questions

1. The area of triangle formed by the points $(-5,0), (0,-5)$ and $(5,0)$ is SEP-21,PTA-2
   (A) 0 sq. units **(B) 25 sq. units** (C) 5 sq. units (D) none of these
2. A man walks near a wall, such that the distance between him and the wall is 10 units consider the wall to be the Y axis. The path travelled by the man
   (A) $x = 10$ (B) $y = 10$ (C) $x = 0$ (D) $y = 10$
3. The straight line given by the equation $x = 11$ is PTA-1, SEP-20
   (A) Parallel to X axis **(B) parallel to Y axis**
   (C) passing through the origin (D) passing through the point $(0,11)$
4. If $(5,7), (3,p)$ and $(6,6)$ are collinear then the value of p is PTA-5, MAY-22
   (A) 3 (B) 6 **(C) 9** (D) 12
5. The point of intersection $3x - y = 4$ and $x + y = 8$ is PTA-2, JUL-22
   (A) (5,3) (B) (2,4) **(C) $(3,5)$** (D) (4,4)
6. The slope of the line joining $(12,3)$ and $(4,a)$ is $\frac{1}{8}$ the value of $'a'$ is PTA-3
   (A) 1 (B) 4 (C) -5 **(D) 2**
7. The slope of the line which is perpendicular to line joining the points $(0,0)$ and $(-8,8)$ is
   (A) -1 **(B) 1** (C) $\frac{1}{3}$ (D) -8 MAY-22
8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of the perpendicular bisector of PQ is PTA-6, JUL-22
   (A) $\sqrt{3}$ **(B) $-\sqrt{3}$** (C) $\frac{1}{\sqrt{3}}$ (D) 0
9. If A is a point on the $y-$ axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is
   (A) $8x + 5y = 40$ (B) $8x - 5y = 40$ (C) $x = 8$ (D) $y = 5$
10. The equation of the line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ PTA-4
    (A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$ **(C) $3x + 7y = 0$** (D) $7x - 3y = 0$
11. Consider four straight lines
    (i) $l_1: 3y = 4x + 5$ (ii) $l_2: 4y = 3x - 1$ (iii) $l_3: 4y + 3x = 7$ (iv) $l_4: 4x + 3y = 2$
    Which of the following statement is true
    (A) l_1 and l_2 are perpendicular (B) l_1 and l_4 are parallel
    (C) l_2 and l_4 are perpendicular (D) l_2 and l_3 are parallel
12. A straight line has equation $8y = 4x + 21$ which of the following is true. PTA-3
    (A) The slope is 0.5 and the y intercept is 2.6 (B) The slope is 5 and the y intercept is 1.6
    (C) The slope is 0.5 and the y intercept is 1.6 (D) The slope is 5 and the y intercept is 2.6
13. When proving that a quadrilateral is a trapezium it is necessary to show PTA-4
    (A) Two sides are parallel **(B) Two parallel and two non- parallel sides**
    (C) Opposite sides are parallel (D) All sides are of equal length
14. When proving that a quadrilateral is a parallelogram by using slopes you must find
    (A) The slopes of two sides **(B) The slopes of two pair of opposite sides**
    (C) The length of all sides (D) Both the length and slopes of two sides
15. (2,1) is the point of intersection of two lines
    (A) $x - y - 3 = 0, 3x - y - 7 = 0$ **(B) $x + y = 3,\ 3x + y = 7$**
    (C) $3x + y = 3,\ x + y = 7$ (D) $x + 3y - 3 = 0, x - y - 7 = 0$

2 mark Questions

1. What is the inclination of a line whose slope is

PTA-3

(i) 0

$m = 0$

$\tan\theta = 0$

Angle of inclination is $\mathbf{0}^\circ$

(ii) 1

Slope $m = 1$

$\tan\theta = 1 \Rightarrow \quad \theta = 45^\circ \ (\because \tan 45^\circ = 1)$

Angle of inclination is 45°.

2. Find the slope of a line joining the points

(ii) $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$

PTA-2

$(x_1, y_1) = (\sin\theta, -\cos\theta)$

$(x_2, y_2) = (-\sin\theta, \cos\theta)$

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\cos\theta - (-\cos\theta)}{-\sin\theta - \sin\theta}$$

$$= \frac{2\cos\theta}{-2\sin\theta}$$

$$= \frac{-\cos\theta}{\sin\theta} = -\cot\theta$$

$$\boldsymbol{m = -\cot\theta}$$

(i) $(5, \sqrt{5})$With the origin (JUL-22)

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{0-\sqrt{5}}{0-5} = \frac{\sqrt{5}}{5}$$

$$\text{Slope} = \frac{\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$$

$$= \frac{\mathbf{1}}{\sqrt{\mathbf{5}}}$$

3. Show that the given points are collinear: $(-3, -4)$, $(7, 2)$ and $(12, 5)$

SEP-21

Let the given points be $A(-3, -4)$, $B(7,2)$ and $C(12,5)$

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{[2-(-4)]}{[7-(-3)]} = \frac{2+4}{7+3} = \frac{6}{10}$$

$$\boldsymbol{m = \frac{3}{5}}$$

$$\text{Slope of } BC = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-2}{12-7}$$

$$\boldsymbol{m = \frac{3}{5}}$$

Slope of AB = Slope of BC

$\therefore$ The given points are collinear.

4. Find the equation of a straight line which has slope $-\frac{5}{4}$ and passing through to the point $(-1, 2)$

MAY-22

Slope $m = -\frac{5}{4}$

Equation of the line passing through the point $(-1,2) \Rightarrow \quad y - y_1 = m(x - x_1)$

$$y - 2 = \left(-\frac{5}{4}\right)(x - (-1))$$

$$4(y - 2) = -5(x + 1)$$

$$4y - 8 = -5x - 5$$

$$5x + 4y + 5 - 8 = 0$$

The required equation is $\mathbf{5x + 4y - 3 = 0}$

5. Find the intercept made by following lines on the coordinate areas.

SEP-21

(i) $3x - 2y - 6 = 0$

$3x - 2y = 6$ Dividing by 6

$$\frac{x}{2} + \frac{y}{-3} = 1$$

x Intercept $\Rightarrow \mathbf{2}$

y Intercept $\Rightarrow \mathbf{-3}$

5 mark Questions

1. Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are $(-4, -2), (-3, k), (3, -2)$ and $(2, 3)$

PTA-5, SEP-20

Area of quadrilateral $= 28$ square units

$$\frac{1}{2}\begin{bmatrix} -4 & 3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix} = 28$$

$$[(-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12)] = 56$$

$$(-4k + 11) - (3k - 10) = 56$$

$$-4k + 11 - 3k + 10 = 56$$

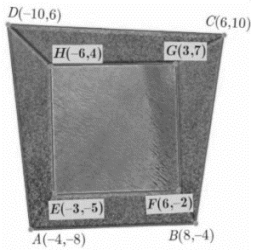
$$-7k = 56 - 21$$

$$-7k = 35$$

$$k = \frac{35}{-7}$$

$$\boldsymbol{k = -5}$$

2. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio. PTA-2



To find the area of patio we have to subtract area EFGH from area of ABCD

Area of ABCD $A(-4,-8), B(8,-4), C(6,10), D(-10,6)$

$$= \frac{1}{2}\begin{bmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{bmatrix}$$

$$= \frac{1}{2}[(16 + 80 + 36 + 80) - (-64 - 24 - 100 - 24)]$$

$$= \frac{1}{2}[212 - (-212)]$$

$$= \frac{1}{2}[212 + 212] = \frac{1}{2}[424]$$

$$= \mathbf{212}\ \textbf{Square units.}$$

Area of EFGH $E(-3,-5), F(6,-2), G(3,7), H(-6,4)$

$$= \frac{1}{2}\begin{bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{bmatrix}$$

$$= \frac{1}{2}[(6 + 42 + 12 + 30) - (-30 - 6 - 42 - 12)]$$

$$= \frac{1}{2}[90 - (-90)]$$

$$= \frac{1}{2}[180]$$

$$= \mathbf{90}\ \textbf{Square units.}$$

Area of the concrete patio = Area of $ABCD$ − Area of $EFGH = 212 - 90 = \mathbf{122}$ **sq.units.**

3. The line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(8,12)$ and $(x,24)$. Find the value of x. PTA-6

Slope of the line passing through the points $(-2,6)$ and $(4,8)$

Slope $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{8-6}{4-(-2)} = \frac{2}{4+2} = \frac{2}{6} = \frac{1}{3} \ldots\ldots\ldots\ldots (1)$$

Slope of the line passing through the points $(8,12)$ and $(x,24)$

Slope $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24-12}{x-8} = \frac{12}{x-8} \ldots\ldots\ldots (2)$

Since these lines are perpendicular to each other

$$m_1 \times m_2 = -1 \Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$$

$$\frac{4}{x-8} = -1$$

$$4 = -(x-8)$$

$$4 = -x + 8$$

$$x = 8 - 4$$

$$x = 4$$

4. A quadrilateral has vertices at $A(-4,-2), B(5,-1), C(6,5)$ and $D(-7,6)$. Show that the mid-points of its sides form a parallelogram.

MAY-22

Midpoint of the side $AB = \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right)$

$= \left(\frac{1}{2}, \frac{-3}{2}\right) = P$

Midpoint of the side $BC = \left(\frac{5+6}{2}, \frac{-1+5}{2}\right)$

$= \left(\frac{11}{2}, \frac{4}{2}\right)$

$= \left(\frac{11}{2}, 2\right) = Q$

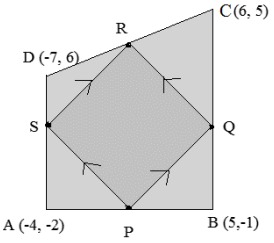
Midpoint of the side $CD = \left(\frac{6-7}{2}, \frac{5+6}{2}\right)$

$= \left(-\frac{1}{2}, \frac{11}{2}\right) = R$

Midpoint of the side $DA = \left(\frac{-7-4}{2}, \frac{6-2}{2}\right)$

$= \left(-\frac{11}{2}, \frac{4}{2}\right) = \left(-\frac{11}{2}, 2\right) = S$

Slope of opposite sides:



Slope of the PQ

$= \frac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$

Slope of RS

$= \frac{2-\frac{11}{2}}{-\frac{1}{2}+\frac{11}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$

Slope of $QR = \frac{\frac{11}{2}-2}{-\frac{1}{2}-\frac{11}{2}} = \frac{\frac{7}{2}}{-\frac{12}{2}} = -\frac{7}{12}$

Slope of $PS = \frac{2+\frac{3}{2}}{-\frac{11}{2}-\frac{1}{2}} = \frac{\frac{7}{2}}{\frac{-11-1}{2}} = \frac{7/2}{-12/2} = -\frac{7}{12}$

$\therefore \quad PQ = RS, \quad QR = PS$

Hence, mid-points of its sides form a parallelogram

5. A cat is located at the point $(-6,-4)$ is xy- plane. A bottle of milk is kept at $(5,11)$ The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

JUL-22

Equation of the path $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$(-6,-4)$ and $(5,11)$ $\quad (x_1, y_1) = (-6,-4)$, $(x_2, y_2) = (5,11)$

$\frac{y+4}{15} = \frac{x+6}{11}$

$11(y+4) = 15(x+6)$

$11y + 44 = 15x + 90$

$0 = 15x - 11y + 90 - 44$

The required equation is $\mathbf{15x - 11y + 46 = 0}$

6. Find the equation of a straight line which has slope $-\frac{5}{4}$ and passing through to the point $(-1,2)$

Slope $m = -\frac{5}{4}$

MAY-22

Equation of the line passing through the point $(-1,2) \Rightarrow \quad y - y_1 = m(x - x_1)$

$y - 2 = \left(-\frac{5}{4}\right)(x - (-1))$

$4(y-2) = -5(x+1)$

$4y - 8 = -5x - 5$

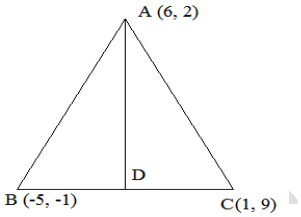
$5x + 4y + 5 - 8 = 0$

The required equation is $\mathbf{5x + 4y - 3 = 0}$

7. Find the equation of the median and altitude of triangle ABC through A where the vertices are $A(6,2), B(-5,-1)$ and $C(1,9)$

SEP-21, PTA-6

The median drawn passing through the vertex A intersect the side BC at the mid point.



$$D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$D = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{8}{2}\right) = (-2,4)$$

$(x_1, y_1) = B(-5,-1)$
$(x_2, y_2) = C(1,9)$

Equation of the median AD:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$(x_1, y_1) = A(6,2)$
$(x_2, y_2) = D(-2,4)$

$$-8(y-2) = 2(x-6)$$

$$-8y + 16 = 2x - 12$$

$$0 = 2x + 8y - 12 - 16$$

$$2x + 8y - 28 = 0$$

$$\div 2, \quad x + 4y - 14 = 0$$

If a line passing through the vertex A is altitude, then it will be perpendicular to BC

Slope of BC $\quad m = \frac{y_2-y_1}{x_2-x_1} = \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

$$m_1 \times m_2 = -1$$

$$\frac{5}{3} \times m_2 = -1$$

$$m_2 = -1 \times \frac{3}{5}$$

$$= -\frac{3}{5}$$

Equation of altitude passing through A

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{5}(x - 6)$$

$$5(y-2) = -3(x-6)$$

$$5y - 10 = -3x + 18$$

$$3x + 5y - 10 - 18 = 0$$

$$\mathbf{3x + 5y - 28 = 0}$$

8. Find the equation of a straight line joining the point of intersection of $3x + y + 2 = 0$ and $x - 2y - 4 = 0$ to the point of intersection of $7x - 3y = -12$ and $2y = x + 3$ PTA-3

$3x + y + 2 = 0$..... (1)

$x - 2y - 4 = 0$..... (2)

$2 \times (1) \Rightarrow 6x + 2y + 4 = 0$

$(2) \Rightarrow \quad x - 2y - 4 = 0$

$7x \qquad = 0$

$x = \frac{0}{7} = 0$

$x = 0$

sub $x = 0$ in (1) we get

$3(0) + y + 2 = 0$

$y = -2$

Point of intersection of the first two lines is $(0, -2)$

$7x - 3y = -12$..... (3)

$2y = x + 3$

$x - 2y = -3$..... (4)

$2 \times (3) \Rightarrow 14x - 6y = -24$

$-3 \times (4) \Rightarrow \quad 3x - 6y = -9$

$(-) \quad (+) \qquad (+)$

$11x \qquad = -15$

$x = -\frac{15}{11}$

Sub $x = -\frac{15}{11}$ in (4) we get

$-\frac{15}{11} - 2y = -3$

$-2y = -3 + \frac{15}{11}$

$-2y = \frac{-33+15}{11}$

$-2y = -\frac{18}{11}$

$y = \frac{9}{11}$

Point of intersection of other set of lines is $\left(\frac{-15}{11}, \frac{9}{11}\right)$

To find the equation of the line passing through the points $(0, -2)$ and $\left(\frac{-15}{11}, \frac{9}{11}\right)$

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y+2}{\frac{9}{11}+2} = \frac{x-0}{-\frac{15}{11}-0}$

$\frac{y+2}{\frac{31}{11}} = \frac{x-0}{-\frac{15}{11}}$

$-15(y + 2) = 31(x - 0)$

$-15y - 30 = 31x$

$\therefore$ The required equation is $\mathbf{31x + 15y + 30 = 0}$

9. The area of a triangle is 5 sq. Units. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex is (x, y) where $y = x + 3$. Find the coordinates of the third vertex. PTA-1

Given, area of triangle ABC is 5 sq. Units and $A(2,1), B(3,-2), C(x,y)$ where $y = x + 3$

Area of $\Delta = \frac{1}{2}\begin{bmatrix} 2 & 3 & x & 2 \\ 1 & -2 & y & 1 \end{bmatrix} = 5$

$(-4 + 3y + x) - (3 - 2x + 2y) = 10$

$x + 3y - 4 - 3 + 2x - 2y = 10$

$3x + y = 17$ ..... (1)

Given $y = x + 3$ sub in (1)

$3x + x + 3 = 17$

$4x = 14 \Rightarrow x = \frac{14}{4} \Rightarrow x = \frac{7}{2}$

Substitute, $x = \frac{7}{2}$ in $y = x + 3 \Rightarrow \quad y = \frac{13}{2}$

$\therefore$ Third vertex is $\left(\frac{7}{2}, \frac{13}{2}\right)$